ABACABAX

A short guide to the reckoning methods of the four families

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The following guides are published as appendices to
"Abacabax"
ISBN: 9798565714739

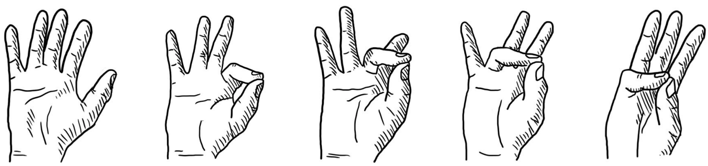
"Abacabax" is available on Amazon.com

Notes from the first family:
A short guide to finger reckoning

Dijin finger reckoning is very quick and easy for adding and subtracting numbers from 0-99. The system is built on connections between 5s and 10s, and two different ways to add or subtract digits depending on the digits' relationships to 5s and 10s.

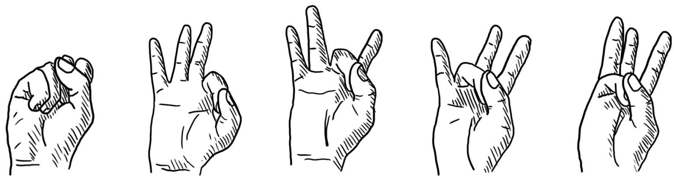
Basic principles

The right hand is used for *ones*, from 0-9, and the left hand is used for *tens*, from 0-90. Zero is an open, empty hand. The numbers 1-4 are made by touching the thumb flatly against the pad of a finger.



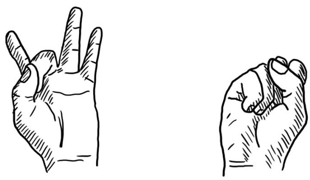
0 1 2 3 4

Five is a closed fist. The numbers 6-9 are made by touching the thumb against the fingernail of a bent finger.



5 6 7 8 9

The left hand uses these same positions to count tens.



75 = 7 on the left hand (70) and 5 on the right hand

Adding and subtracting digits

If the thumb is already touching a finger, changing the position of the thumb from one finger to another changes the value on the hand by the number of positions moved. Moving the thumb two positions for example adds or subtracts 2 to the value in the reckoning.

Bending or straightening a finger so that the thumb moves from one side of that finger to the other changes the value by 5. For example, to add 2+5, the thumb starts against the pad of the middle finger to show 2. Bending that middle finger so the thumb slides to the fingernail side of the finger adds 5 to make 7.

This movement from one side of a finger to another is an anchoring movement for adding or subtracting numbers other than 5. To add 6, for example, the thumb is moved to the other side of the finger it is touching and then its position is incremented one position. When learning, it is helpful to do these two moves separately as shown below, but with practice these two moves blend into one. This move may or may not be accompanied by a change on the left hand depending on whether or not the addition or subtraction affects the number of tens. This will be discussed shortly.

2+6: start 2 +5, thumb changes sides +1 = 8

Adding 4 can be accomplished by first adding 5 and then moving one position upwards to subtract 1.

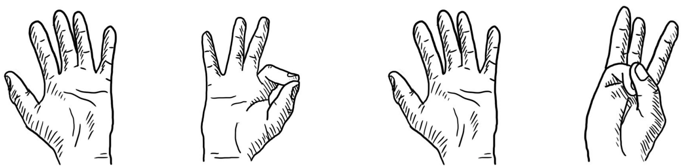
4+4: start 4 +5, thumb changes sides –1 = 8

Adding 7 or 3 can involve adding 5 and then incrementing 2 positions downwards (for +7) or upwards (for +3). Other numbers can be anchored in such a manner to 5, although for some operations it is easier to anchor the movements to 10 instead.

Two ways to add

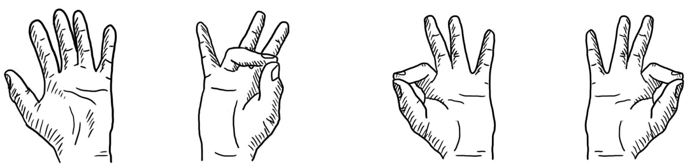
There are two different ways to add a new number to the number currently on the fingers. Which of the two methods is used depends on what number is showing. We will use +8 as an example.

If the right hand is showing 1, to add 8 the right thumb moves +5 and then moves 3 positions downwards for a combined 5+3 as shown here:



Right and left hands with 1 +8 = Right +5 and +3, result 9

However, if 2 can be easily subtracted from the right hand, adding 8 is done as –2 on the right hand and +10 on the left hand. For example when the right hand show 3, adding 8 is accomplished as +10 –2 as shown here.



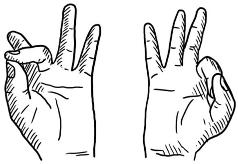
3 +8 = Left +10, right –2, result 11

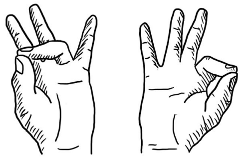
Likewise, adding 7 can be accomplished with –3 and +10. Adding 6's are most often done as +5 and +1, and adding 9's are most often done as –1 and +10.

Incrementing 10s when adding

Any time during an addition when the thumb travels from the back (fingernail) side of the fingers to the front (pad) side of the fingers the left hand adds +10. This will become automatic with practice.

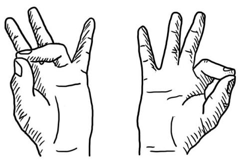
For example, here are two hands showing 26. When 5 is added, the thumb moves from the back of the first finger to the front of the first finger, so the left hand increments from 20 to 30. The result is 31.

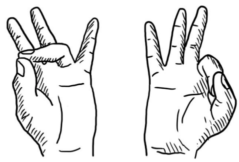




Start 26 +5 = Right thumb moves back to front, so left +10. Result 31.

If 5 is then added again, this time the thumb moves from the front to the back so the left hand is not incremented.

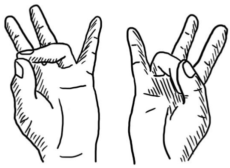




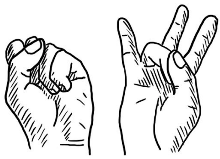
Start 31 +5 = Right thumb moves front to back, so left no change. Result 36.

Multi-digit addition

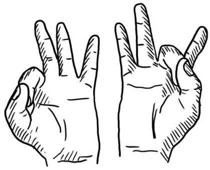
With two-digit numbers, the digits are added separately to the appropriate hands. With 38 + 29, the hands start with 38. Add 20 on the left hand, then add 8 on the right hand, as a –2 on the right and +10 on the left. The result is 67.



Start 38



Left hand +20



Left hand +10, right –1

Subtraction

All movements for subtraction are the opposite of addition. Adding 4, for example, is done with a +5 and –1, while subtracting 4 is –5 and +1. Adding 8 is usually done with +10 and –2, while subtracting 8 is usually –10 and +2.

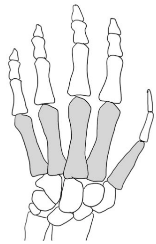
When performing addition, when the thumb moves from the back to the front the 10s are incremented on the left hand. When subtracting, the opposite is done: when the thumb moves from the *front* to the *back* and the left hand moves to *subtract* 10. For example, with 22 – 5, the right thumb moves from the front to the back of the middle finger (a change of 5), which the left thumb moves from the 20 position to the 10 position. The result is 17.

These movements will become natural with practice.

Other operations

Finger reckoning is very quick but weak compared to other reckoning methods due to the limited number of digits and general use of only addition and subtraction. Some Finger Reckoners have been able to extended the art to include other operations and to increase the number of digits by using other parts of the hands or body. Details are unavailable due to Finger Reckoners' aversion to sharing techniques, even amongst themselves.

Notes from the second family: A short guide to bone throwing



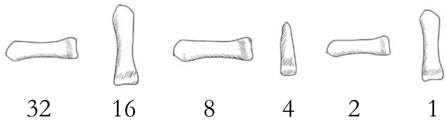
Bone-Throwers use numbers represented as powers of two. This makes operations quick and simple, though the unfamiliarity of the number system can make setting up the numbers more challenging. Bone-Throwers calculate with ten metacarpal bones from a pair of human hands. If hand bones are not readily available, small twigs or toothpicks may be used instead for practice.

Basic principles

Bones are placed in a line. The line may be as long as the number of bones available for use, usually ten. Bones can be placed “fallen down” in a horizontal position or “standing up” in a vertical position. (Bones that are “standing up” are not literally standing and balancing on one end, they are simply turned so they are in a different direction.) Any bone lying down has a value of zero. The basic starting position is with all bones lying down and showing values of zero.

When a bone is turned so it is standing (in a vertical position), it has a value according to its position. The bone furthest to the right has a value of 1 when turned to the vertical position. The second bone from the right assumes a value of 2, the third bone a value of 4, the fourth a value of 8, and so on. The values double from one position to the next.

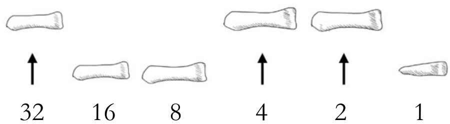
The following illustration shows the number 21 with six bones. The standing bones have values 16, 4 and 1. Together their sum is 21.



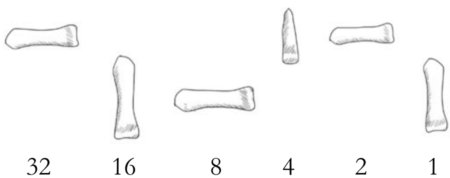
With these six bones, all numbers from 0 to 63 may be formed, each in a unique arrangement. With ten bones, the limit is much higher – numbers can be created up to 1023.

A second way to represent a number

Instead of standing or fallen bones, numbers are also representing by raising or lowering bones. Here, for example, the number 38 is made by raising the bones in the 32, 4 and 2 positions (32 + 4 + 2 = 38):



Both methods of representing numbers can be used at the same time to show different numbers on one set of bones. This is useful for operations where the two numbers to be operated on can be set up simultaneously. In the following illustration, the standing bones represent 21, and the raised bones represent 38.



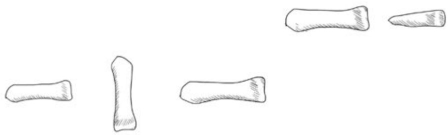
Addition

Adding numbers is accomplished by setting up both numbers to be added (as shown above) and then sliding down the raised bones, turning them as they are moved. This move may or may not be followed by some additional turns of bones, depending on whether the raised bone stands up or falls down when lowered. Here are two simple examples.

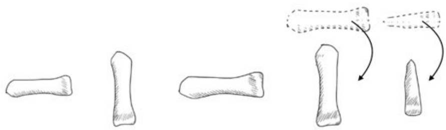
Example with 8 + 3

First set up the bones to show 8 by standing the bone in position 8.

Now set up the second number. Raise the bones in positions 2 and 1 to show 3.



Slide down the raised bones one at a time, turning each from a fallen to a standing position. Because the bones are stood up as they are lowered, no extra move is required for each step.



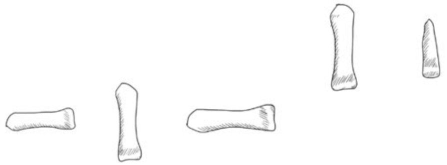
The bones now show the result of 8 + 3: 11.

Falling bones

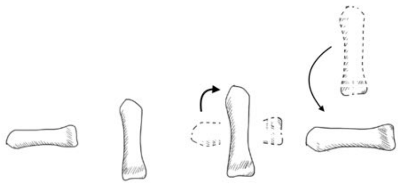
If a raised bone is standing up, it will fall to a horizontal position when it is lowered. Bone-Throwers say that addition is a process of building and no one falls without another standing up. Thus, when a raised bone falls down as it is lowered back into the line of bones, the bone to the *left* is turned. If this bone stands, the step is complete. If this bone also falls down, the next bone is turned and so on, until a bone stands. Each step of the calculation is not complete until exactly one bone is turned to a standing position.

Example with 11 + 3

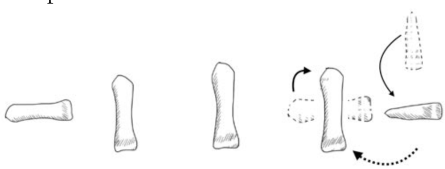
The following bones are set up to calculate 11 + 3. The standing bones are in positions 8, 2 and 1 (8 + 2 + 1 = 11), and the raised bones are in positions 2 and 1 (2 + 1 = 3).



Step 1: Working from the left, lower the first raised bone and turn it. The bone has now fallen down and so the bone to the left is turned, causing it to stand and completing the step.



Step 2: The second raised bone is lowered and turned to a fallen position, so the bone to the left is turned. This bone is now standing, so the step is complete.



All bones are now back in the line, and the result of the addition, 14 (8 + 4 + 2) is revealed.

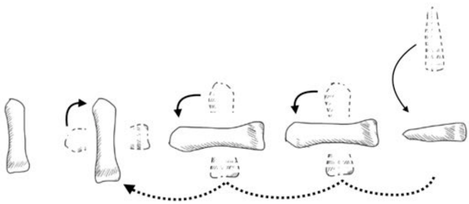
Chain reactions

When a bone is turned so that it falls over it may start a chain reaction if there are several bones in a row that are standing, each falling bone causing the next to fall in a domino-like effect. (If a chain reaction extends beyond the left end of the row of bones during addition, the result has exceeded the limit of the number of bones and will not work without introducing an additional bone.)

Example with 23 + 1

Here's a simple example with 23 + 1. The standing bones are 16, 4, 2 and 1 which represents 23. The bone in position 1 is raised, representing the number to be added (which is simply 1 in this example).

Sliding the raised bone down starts a chain reaction. Since the bone is turned so that it falls over, the bone to the left must be turned. This causes that bone also to fall over, so the next bone to the left must also be turned. That bone falls over as well, so the chain reaction continues to the next bone. Finally the bone is position 8 is turned to a standing position the step is completed.



The result of the operation is 24 (16 + 8).

These examples have all used small numbers. Traditional bone throwing uses 10 bones, and so finding sums such as 683 + 259 is nearly as quick as finding these smaller sums. A challenge for those not raised as Bone-Throwers is the ability to quickly recognize and create numbers using powers of 2. This is a skill that develops quickly with practice.

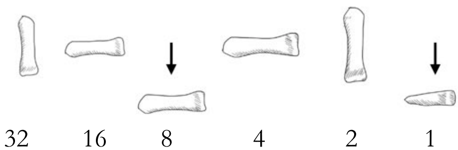
Subtraction

One of the greatest mysteries in Bone Thrower reckoning is that subtraction uses the exact same moves as addition. The first number in the operation is set up on the bones using standing and fallen bones as usual. When entering the second number, however, instead of raising the bones to create the number, these bones are *lowered* instead.

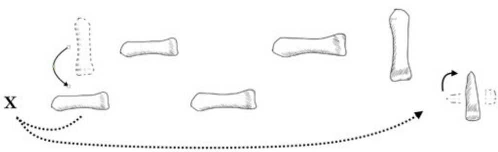
The bones are now operated on just like while doing addition: lowering and turning the bones in the higher position. There is one small difference: when a chain reaction extends past the left end of the line of bones, the chain reaction "wraps around" and continues on the right side of the line of bones, proceeding as usual until a bone is turned to a standing position and the reaction ends. Surprisingly, this wrap-around will always happen *exactly once* during every subtraction, a phenomenon no outsiders have been able to explain.

Example with 34 – 9

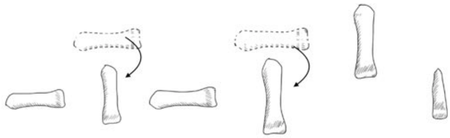
The standing bones show the greater number (34 = 32 + 2). The bones representing 9 (9 =8 +1), are lowered *downwards.*



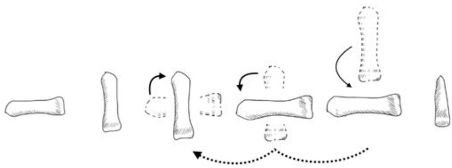
Step 1: The first bone is lowered and turned to a fallen position. This should cause the bone to the left to be turned, but there are no bones to the left. The maneuver wraps around to the right side of the line of bones and continues. The rightmost bone stands, completing the step.



In steps 2 and 3, the next two raised bones are lowered and turned to standing positions. This does not affect any other bones.



In step 4, the final raised bone turns to a fallen position as it lowered, starting a short chain reaction.

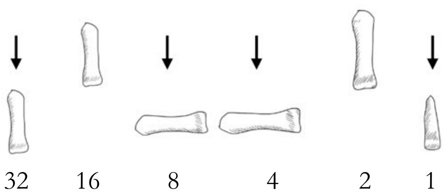


The bones now show the result: 34 – 9 = 25 (16 + 8 + 1).

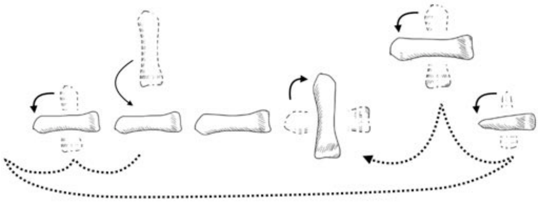
Example with 51 – 45

Here is one more example to show a lengthy chain reaction that wraps around and affects both raised and lowered bones.

51 is set up by standing up bones 32, 16, 2 and 1 (51 = 32 + 16 + 2 +1). To make 45, bones are lowered in positions 32, 8, 4 and 1 (45 = 32 + 8 + 4 + 1).

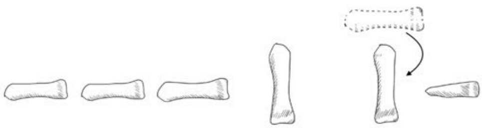


The first step sets off a long chain reaction. The first raised bone is lowered and falls down. The bone to its left also falls over, so the reaction continues, wrapping around to the right side where the rightmost bone is turned. This bone, too, falls over, as does the bone in the 2 position. Finally the bone in the 4 position stands up, ending the reaction.



Note that during the reaction the bone in position 2, which is raised, is *not* lowered – a chain reaction only affects whether bones are standing up or fallen and does not affect their raised or lowered positions.

In step 2, the final raised bone is lowered and stood up, completing the step.



The result of the operation is now revealed: 51 – 45 = 6.

It is perhaps surprising that addition and subtraction use the same method (apart from the wrap-around chain reaction). When adding, the number to be added is raised and when subtracting the number to be subtracted is lowered, and then the same algorithm is applied. Little is known of the Bone-Throwers and why these complementary methods function is another of these mysteries.

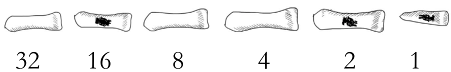
Multiplication

Conceptually, Bone-Thrower multiplication is easy, but the process can be confusing with traditional bones. Bone-Throwers mark their bones on one side with burn marks to make multiplication much simpler. These marks allow a *third* way to encode a number on the set of bones, a practice called "double two-ness."

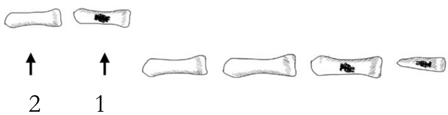
To multiply, a number is added one or more times to the line of bones *in different positions*. These positions are determined by the marks on the bones in the initial set up.

Example with 19 × 3

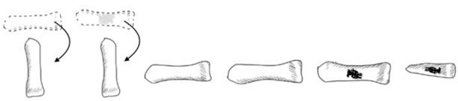
We will demonstrate with 19 × 3. Start with the bones in a line showing 19 with the markings in the 16, 2 and 1 positions.



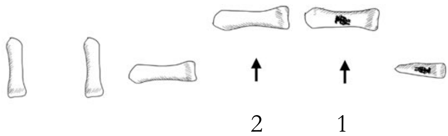
Working from left to right, the first mark will temporarily be considered as the 1 position. Raise bones to represent the number 3, using the mark as a new reference point for the temporary 1.



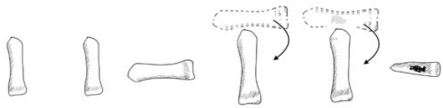
This number is now added in the usual manner. The marked bone may now be flipped over so the mark is hidden.



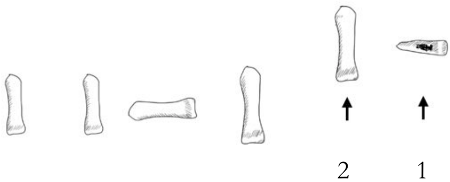
The next marked bone is now considered the new 1 position. Raise bones to again represent 3 with the new position for 1.



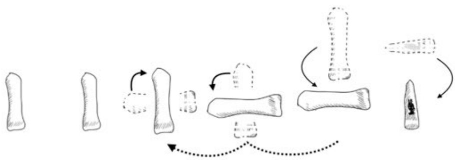
This number is added and the marked bone can be flipped.



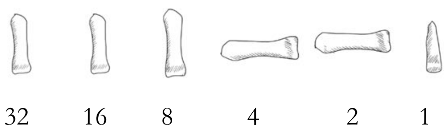
There is one marked bone remaining. Use this as a reference for 1 and raise bones to represent 3.



Complete the addition. In this example, there is one two-step chain reaction.



The marked bone can be flipped. The final result is revealed. Bones 32, 16, 8 and 1 are standing, so 19 × 3 = 57.

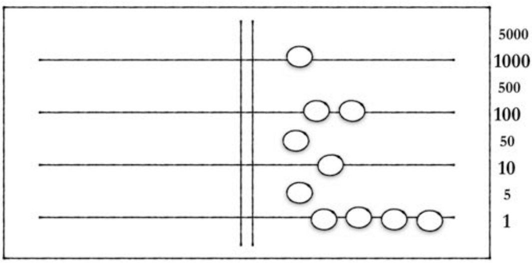


Notes from the third family:
A short guide to calculating on the abax

To "calculate" literally means to reckon with stones. Calculists use small rounded stones called "abac" on stone tablets called "abaxes" to organize calculations efficiently.

Abaxes are sized to fit comfortably on the left forearm, held with the left hand gripping the right edge and abax cradled securely against the user's elbow and body. Calculists have pockets with large openings sewn over the left breast of their shirts to hold abac, making it easy to take out or return stones as needed.

A standard abax has four lines carved horizontally and a double line carved vertically dividing the board in two. Numbers are created by placing stones on or between the horizontal lines. Stones on the lowest line have a value of 1. Lines above increase in value 10 times per line, so the other lines have values 10, 100 and 1000. Stones are also placed above the lines to have values 5 times that of the line below, or 5, 50, 500 and 5000 at the very top.



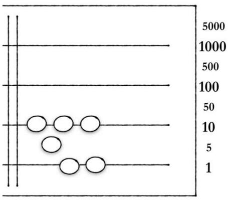
In the picture above, the number on the right side of the board is 1269 = 1000 + 2 × 100 + 50 + 10 + 5 + 4 × 1. (Note: abaxes are usually not marked with numbers. Line values are included in these illustrations for clarity.)

Lines (or spaces between lines) may contain as many stones as there is space for, and where a stone is placed on a line is not important for addition or subtraction. Groups of stones are traded throughout calculations and at the end of the calculation, so that 5 stones on the 10 line for example can be traded for 1 stone on the 50 line, 2 stones on the 500 line can be traded for 1 stone on the 1000 line, and so on.

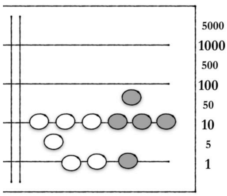
Addition

Nothing could be easier than Calculist addition. Simply place stones on the board to represent the first number, place additional stones on the same lines to represent the second number, and then make appropriate exchanges to minimize the number of stones needed to represent the result. (Beginners sometimes build each number on the two sides of the board and then slide them together, but this extra step is generally not needed.)

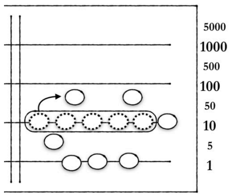
Example with 37 + 81



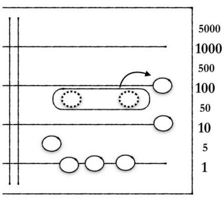
Set up 37



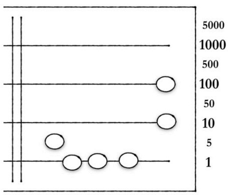
Add 81



Regroup
five 10s = one 50



Regroup, two 50s for one 100



Final result: 118

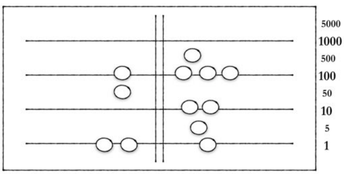
Subtraction

To subtract, the two numbers to be operated on are placed on either side of the board. The larger number is placed on the right, and the number to be subtracted is placed on the left. During the operation, equal numbers of stones are removed from both sides of each line (including spaces between lines) until the left side is empty.

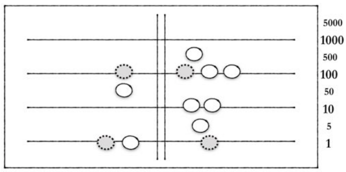
If there are not enough stones on the left side of a line to match the stones on the right side, it is necessary to exchange groups of stones on the right side. For example if a 100 stone is to be removed but the 100 line is empty on the right side, an exchange of a 500 for five 100s or some other exchange is made to bring enough stones to at least match the number on the right.

Subtraction is very flexible. All exchanges may be made at the start or during the process, and equal groups of stones may be removed from both sides of a line at any time and in any order.

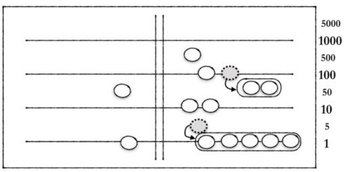
Example with 826 – 152



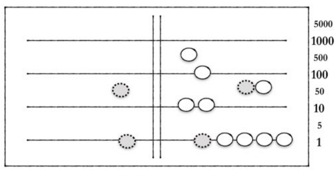
152 is on the right, 826 on the left.



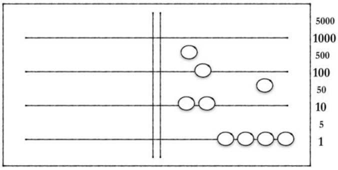
A pair of 100s and a pair of 1s can be removed immediately.



There is no match for the 50 or the 1 on the right. Exchanges must be made on the left.



The 50 and the 1 on the left can now be paired with matching stones on the right and removed.



The left side is empty, the right side
holds the result. 826 – 152 = 674.

Multiplication

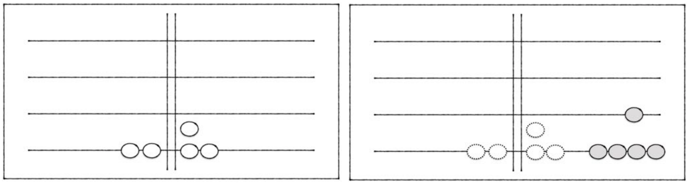
To multiply, values on lines (the digits) are multiplied together several times until every combination of a digit on the right and a digit on the left has been performed. Single digit multiplication is done mentally, so a Calculist must have previously memorized multiplication facts up to 9 × 9. The results of each mental calculation are placed in appropriate places on the abax, which holds these partial results as they are combined to find the result of an operation with much larger numbers.

Determining these positions where the partial results are placed is important and while the principle is not difficult, it requires a little experience and practice to master.

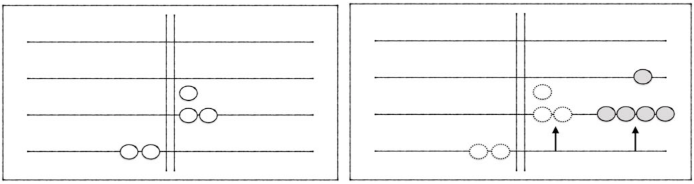
Examples of different placements

All of the calculations in these next four examples are simple enough that an abax is not needed, but they will demonstrate principles used in more complex computations, namely that the location of partial results depends on the locations of the two digits being multiplied.

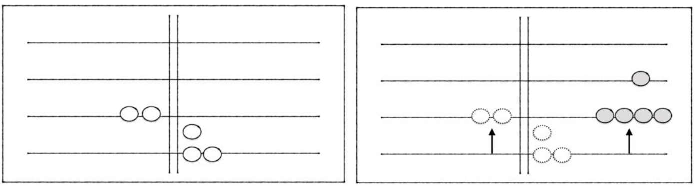
2 × 7. The numbers are set up on each side of the board, close to the center line. The product, computed mentally, is placed on the far right of the board. Both the 2 and 7 are on the bottom line, so the result is also on the bottom line.



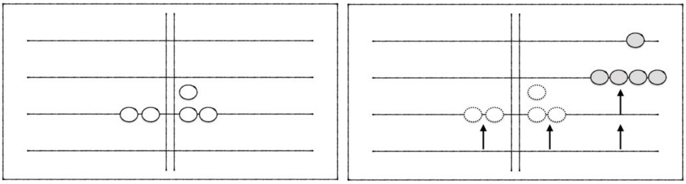
2 × 70. The same digits, 2 and 7, are multiplied to get 14. Since the 7 is raised one line (= 70), the result of 14 is also raised to the same line (= 140).



20 × 7. Because the 2 is raised one line, the 14 is also raised one line. It is helpful to think that results are placed on the line that is the same level as the digit on the right, and then raised by the same amount the number on the left is raised.



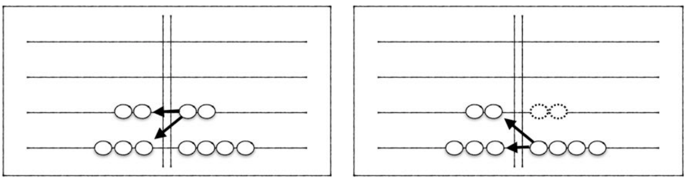
20 × 70. The 2 and the 7 are multiplied to get 14. To find the line where the 14 is placed, start on the line where the 7 is placed. The digit on the left is raised one, so the 14 is raised one line from that starting position. The result is 1400.



Multi-digit multiplication

In order to multiply larger numbers, place the two numbers to be multiplied on the two sides of the abax, near the center line. The top digit on the right (from 1-9) is multiplied with each digit on the left from the top down. Each result is placed on the right side in accordance the placement rules above.

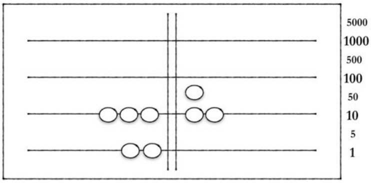
When the top right digit has been multiplied by all digits on the left, the top right digit is now longer needed and these stones are removed. The next digit down on the right is now used for multiplying the digits on the left. This digit on the right will then be removed. The process continues until the original number on the right is completely removed and only the result of the multiplication remains on the right side of the abax.

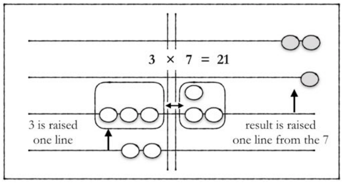


Stones should be regrouped whenever possible to keep the number of stones on the abax to a minimum.

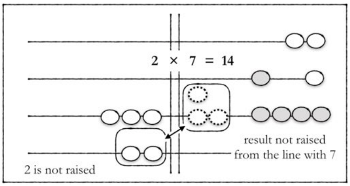
Example with 32 × 70

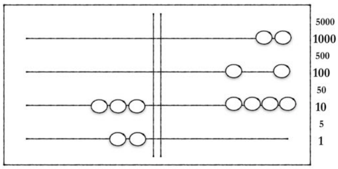
The numbers are set up on different sides of the abax close to the center line. The top two digits, 3 and 7 are multiplied. The result is 21 (shown in gray on the figure below). Since the 3 is raised one line on the abax, the 21 is placed one line *above where the 7 is located* on the right. This “21” has a value of 2100.





The next digit down on the left, 2, is multiplied by the 7. The result of 14 is shown in gray. Because the 2 is not raised above the bottom line, the 14 is placed on the same line as the 7 and is not raised any further. Its value in the calculation is 140.





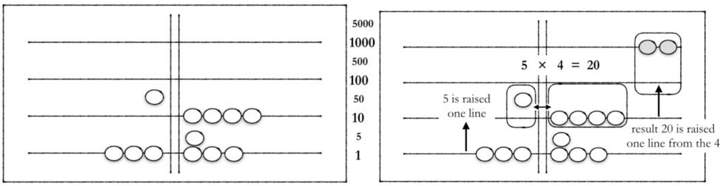
The 7 has now been multiplied by all digits on the left. The 7 is removed. There are no more digits on the right to multiply so the operation is complete. The result is shown on the right: 32 × 70 = 2240.

Example with 53 × 48

Caution! In this calculation, multiplication by the 5 produces partial results that end in a 0. This requires extra care when placing the result.

(a) 53 and 48 are placed on the abax, close to the center line.

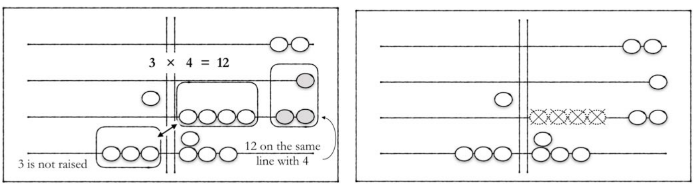
(b) The top digits are multiplied, 5 × 4 = 20. The 20 is placed one line up from the 4 on the left. Notice that the 20 takes up two lines with 0 on the lower line and 2 on the upper line, so the bottom line of the 20 is *empty*. It is easy to make the mistake of placing a *2* one line up instead of placing a *20* one line up, so care must be taken. The partial result, 2000, is the product of 50 and 40.



(a) (b)

(c) The next digit down on the left is multiplied by the top digit on the right. 3 × 4 = 12. Since the 3 is not raised, this 12 is placed on the same line as the 4 and not raised again as in the previous step.

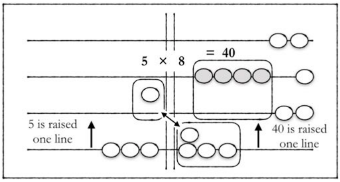
(d) The top digit on the right (4) has now been multiplied by all digits on the left and is removed, leaving more space for the next steps.



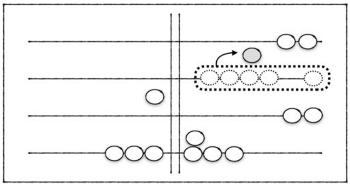
(c) (d)

(e) The top digits are multiplied, 5 × 8 = 40. Again, be careful with the placement, one line above the line with 8 is the start of the 40 – a 40 has 0 stones on the lower line.

(f) There are now 5 stones on the 100s line. These are traded for 1 fifty.



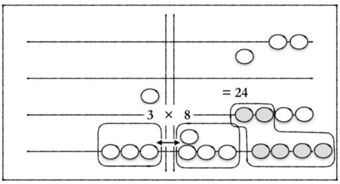
(e)



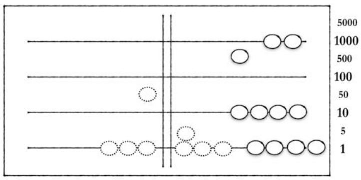
(f)

(g) The next digit down on the left is multiplied by the top digit on the right. 3 × 8 = 24. The 24 is placed on the bottom line.

(h) The top digit on the right can be removed. This is the final digit so the operation is complete. All stones can be removed from the left as well, if desired.



(g)



(h)

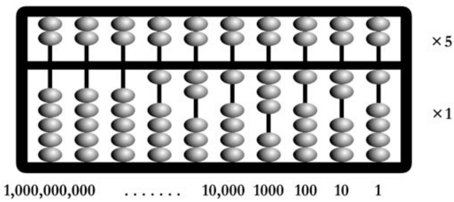
The final result is revealed on the abax: 53 × 48 = 2544.

A standard abax with four lines can handle all 2-digit by 2-digit multiplications, and some 3-digit by 2-digit multiplications (if the first digits are not too great).

Notes from the fourth family: A short guide to the abacus

The abacus is an elegant and powerful tool. It developed directly from the abax and uses many of the same principles. Instead of placing and removing stones, an abacus uses columns of beads that slide up and down on rods, allowing for extremely quick calculations with very large numbers. Further, the abacus offers the convenience of a single device with moving parts all held together in a single frame that fits easily in a pocket.

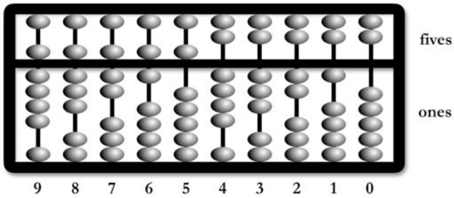
An abacus has several rods with beads divided in two sections by a crossbeam. The top section has two beads that have a value of 5, and the bottom section has five beads with a value of 1. (Some abacuses have 1 and 4 beads, and some have 1 and 5).



The rods represent increasing powers of 10 from the right, so numbers can be set up as we are accustomed to seeing. The number of rods can vary, in the examples here we will use a standard Abacist abacus with 10 rods.

The abacus is cleared by sliding all the beads on the top upwards, and all of the beads on the bottom downwards. Digits are formed by sliding beads towards the crossbeam. This is usually done with a pinching motion so that 1 beads and 5 beads are moved simultaneously.

The ten different digits are shown below, each on a separate rod.

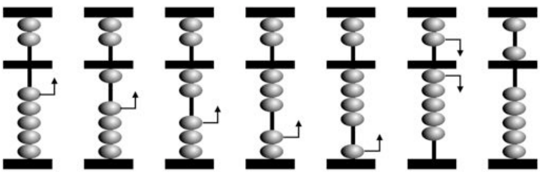


Addition on the abacus is very quick. It requires learning two different moves for placement of each digit. After practicing addition separately with each of these digits as described below, adding numbers of any size can be done very easily and almost automatically.

Adding 1s and trading 5

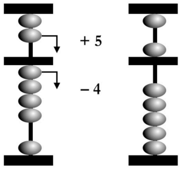
Begin with an empty abacus. 1 is added to the first rod (the rod on the right) by sliding 1 bead upwards with the right thumb. Continue adding one until all 5 one-beads are up.

When all 5 beads in the lower portion of the abacus are up, they are immediately traded for a five-bead. The trade is done in one movement, with the thumb sliding down all 5 one-beads while the index finger slides down 1 five-bead. Trades are always done by moving beads in the same direction.



Counting 0 to 5, with a trade

Notice that when adding 1 to a rod that is showing 4 it is not necessary to raise 1 bead and then lower all 5. This extra motion can be eliminated by instead sliding the 4 beads back downwards and sliding down a five-bead at the same time, in effect performing a –4 and +5 at the same time to give the result +1.



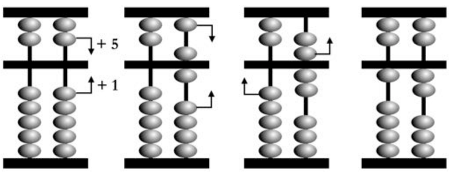
Adding one is done by (a) sliding up 1 one-bead, or (b) sliding down 4 one-beads on the bottom and 1 five-bead on the top if there are 4 beads available to slide downwards.

Likewise, every number added to a rod can be done in two different ways, determined by what number is already on the rod.

Adding 6s and trading 10

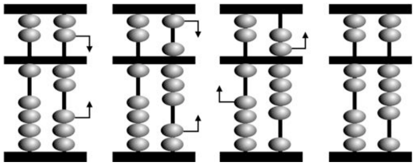
Starting with a cleared abacus, add 6 to the first rod. This is done with a pinching motion to bring down 1 five-bead and bring up 1 one-bead. Repeat this action to add another 6 to the same rod.

There are now 2 five-beads on the rod – these are to be immediately traded for 1 ten-bead. The 2 five-beads are raised with the index finger at the same time that 1 ten-bead is raised with the thumb. Notice that like the trade with 1s and 5s, all of the beads move in the same direction.



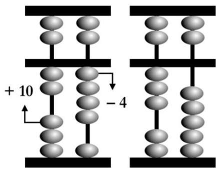
0 + 6   + 6   trade   = 12

Continue adding 6s with a pinching motion. After 2 more 6s are added, another trade is necessary, 2 five-beads for 1 ten-bead. The abacus now shows 24.



12 + 6 + 6 trade = 24

Adding 6 and adding 1 have a common trait: when there are 4 beads available to be moved down, a different move is used to add these numbers. For +6, the 4 one-beads will moved down with the index finger while 1 ten-bead will moved up with the thumb with a twisting motion. Performing a –4 and a +10 gives a result of +6.



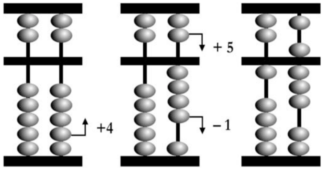
24 + 6 = 30

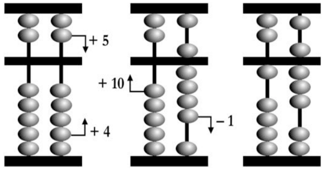
Adding 4s and 9s

To add 4 to an empty rod, 4 beads are raised at the same time with the right thumb. If the rod already has at least 1 one-bead that is already up, instead the move is +5 and –1, the index finger sliding down 1 five-bead while the thumb slides down 1 one-bead.

To add 9 to an empty rod, a pinching motion is used to bring 1 five-bead and 4 one-beads to the crossbeam. If there is at least 1 one-bead available to be moved down, adding 9 is done by lifting 1 ten-bead and lowering 1 one-bead at the same time with a twisting motion.

These two operations, with the two methods of adding each, are shown below.



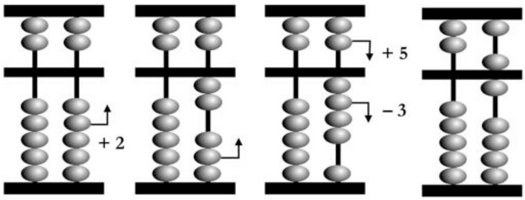


0 + 4 + 4 = 8 0 + 9 +9 = 18

+4 and +9 are related in that if there is at least one bead available to be lowered, the second method of adding these numbers is used.

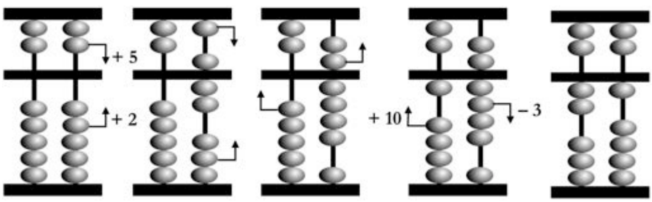
Adding 2s and 7s

Adding 2 involves either sliding 2 one-beads upwards, or if 3 or more one-beads can be lowered then +2 is done as a combination +5 and –3. Both are shown here as we add 2 three times to make 6.



0 + 2 + 2 + 2 = 6

Adding 7 is related to 2. It is accomplished by pinching a five-bead with 2 one-beads if more than 2 beads are available to be lifted, otherwise +7 is a twisting motion to lower 3 one-beads and bring up 1 ten-bead. The series below illustrates both methods of adding 7.



0 + 7 + 7 trade + 7 = 21

Adding 3s and 8

When adding 3 or 8, if 3 lower beads are available to move up they are lifted, either by themselves (for +3) or lifted while a five-bead is lowered (for +8). Otherwise, +3 is accomplished by +5 and –2 simultaneously, and +8 performed as +10 and –2 at the same time.

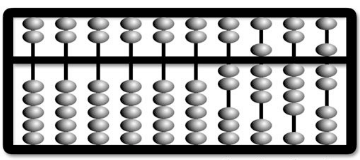
Practicing

Adding each of the digits repeatedly should be practiced many times. For example, add 6 repeatedly until a target number such as 300 is reached. By practicing in such a manner it becomes easy to add a digit to any rod regardless of what digit is already on that rod and perform exchanges quickly and naturally. These moves are essential for other operations.

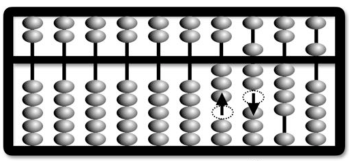
Multi-digit addition

When the moves for adding each separate digit are mastered, multi-digit addition is very easy. The first number is entered on the abacus, then the digits of the number to be added separately on each of the appropriate rods.

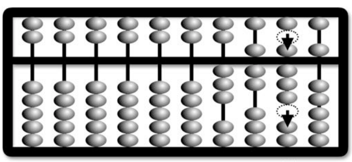
Here is an example with 2845 + 946. One trade is required on the final step of the calculation.



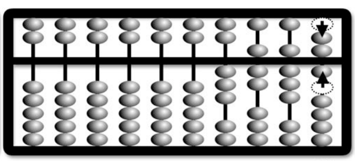
2 8 4 5



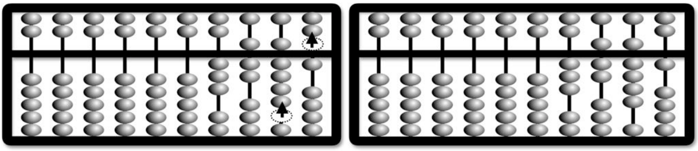
+9



+4



+6



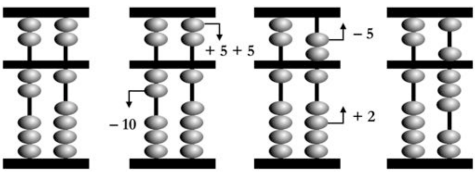
When completed, the abacus shows the result: 2845 + 946 = 3791.

Subtraction

Learning subtraction requires practice subtracting each of the digits as was done above with addition. Each of the moves for subtraction is the exact opposite of the moves in addition. For example, +9 is usually done as a twist with +10 and –1. Likewise, –9 is usually a twist in the opposite direction with –10 and +1.

Trading may need to be done before a digit is subtracted. If –3, for example, is to be performed on a rod with less than a value of 3 on that rod, first 1 ten-bead is traded on the next rod for 2 five-beads on the current rod so there are enough beads available to subtract from.

Here is an example with trading before subtracting. 3 is to be taken from 21 and the rod to be subtracted from has only 1 bead. First 1 ten-bead is traded for 2 five-beads. Now 3 can be subtracted as
–5 and +2.



Advanced abacus users might instead perform –10 and +7 for the above operation and skip the additional step, but it is enough at the start to learn just different sets of moves to be able to use an abacus effectively.

Practice subtraction with each digit separately. Begin with 100 times the digit to be practiced and subtract that digit 100 times.

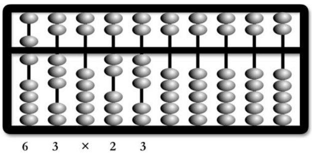
Multiplication

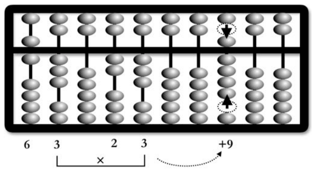
Multiplication is perhaps best understood with a demonstration of 2-digit by 2-digit multiplication. It will help to keep the following "mantra" in mind:

"Last 2, skip 2. First and last, skip 1. Clear the last."

This step sequence will be repeated until the operation is complete. We will demonstrate with 63 × 23. Set up 63 and 23 on the left side, with one empty rod between.

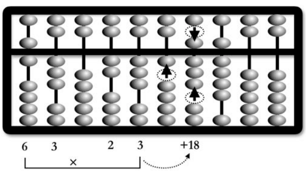
"Last 2, skip 2": Multiply the last two digits mentally (3 × 3 = 9), skip over <u>two</u> rods completely and enter the 9 as shown here.

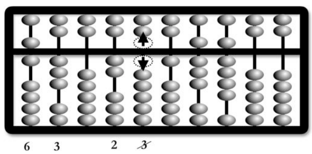




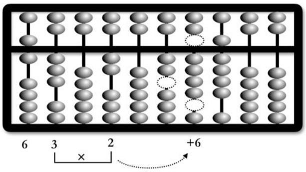
"First and last, skip 1": Multiply the first and last digits (6 × 3 = 18). Skip over <u>one</u> rod and enter the 18 starting on this rod.

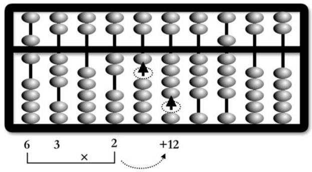
"Clear the last": The last digit (the 3 in 23) is now cleared.



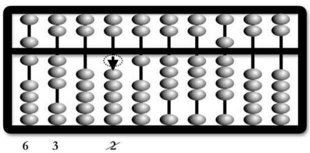


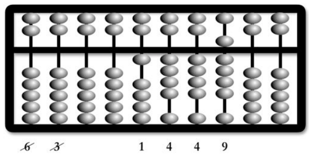
Repeat these same steps. “Last 2, skip 2.” Multiply 3 ×2, skip two rods and enter 6. “First and last, skip 1.” Skip one rod and enter 6 × 3.





“Clear the last.” We are done with the 2 and its rod is cleared. The calculation is complete and original number can also be cleared.





The result is shown: 63 × 23 = 1449.

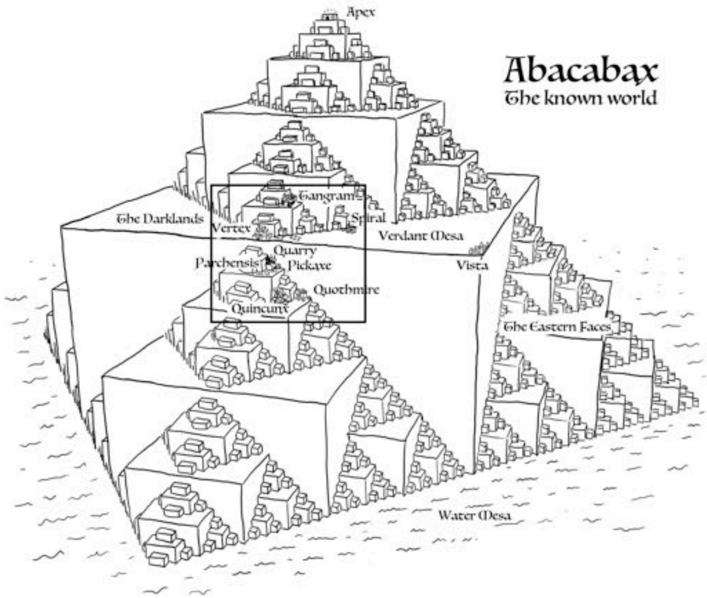
Multiplying with other digit lengths

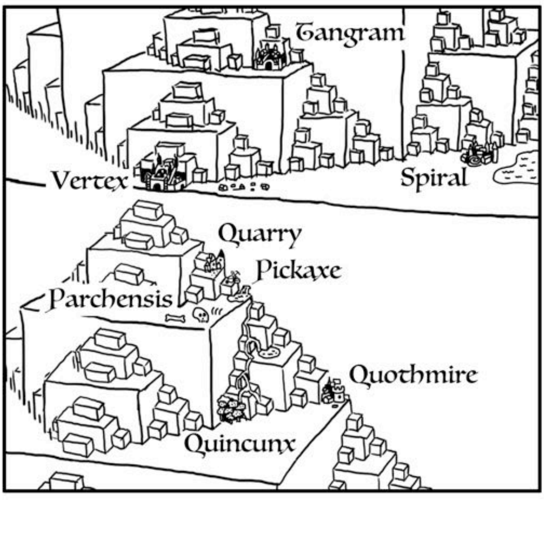
The sequence of steps, “Last 2, skip 2. First and last, skip 1. Clear the last,” works for multiplying a two-digit number by a number of *any* length, as long as the two-digit number is placed to the left. Each time the sequence is completed, one digit is erased from the second number, so the sequence will be done a number of times equal to the number of digits in the second number.

If the first number has a different number of digits than two, the sequence is modified so that the starting “skip” number is equal to the first number’s number of digits. This will ensure there is enough space for the partial results. Multiply each digit of the first number by the last digit of the second number, starting with the last digit of the first number and working through the digits to the left. The skip number decreases with each digit multiplication.

For example, if the first number to be multiplied has three digits, the steps are modified to: “Last 2, skip 3. Middle and last, skip 2. First and last, skip 1. Clear the last.” This same sequence is executed until the second number to multiplied is cleared, regardless of the length of the second number.

The number of rods needed for multiplication depends more heavily on the number of digits in the first number than in the second, because the number of rods to be skipped at the start in each sequence is equal to the number of digits in the first number. With a 10-rod abacus, 2-digit by 5-digit multiplication is possible, but 5-digit by 2-digit is not possible with the above method. Thus, the shorter number should be placed on the left to maximize computing power.


Abacabax
The known world
Apex
Tangram
Spiral
The Darklands
Vertex
Verdant Mesa
Quarry
Parchensis
Pickaxe
Vista
Quothmire
Quincunx
The Eastern Faces
Water Mesa


Tangram
Vertex
Spiral
Quarry
Pickaxe
Parchensis
Quothmire
Quincunx